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PROPAGATION AND ARREST OF AN EDGE CRACK IN A SEMI-INFINITE SOLID

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Abstract—The reflectionless stress intensity factor is determined for an edge crack in a semi-infinite solid, subject to various applied loadings. By comparing its value with that of the static stress intensity factor, the simple K_{Ia} crack arrest procedure is shown to be conservative for this particular situation, in that it over-predicts the crack arrest length. The results are discussed in relation to the water-cooled reactor pressure vessel LOCA problem, which is currently analyzed via the ASME Section XI Code provisions, which are based on the K_{Ia} approach.

INTRODUCTION

The current ASME Code procedure[1] for predicting crack arrest in nuclear pressure vessels is based on a static linear elastic fracture mechanics (LEFM) analysis: the arrest value K_{Ia} of the crack tip static stress intensity factor K_I^{ST} is assumed to be a material property and is referred to as the arrest toughness. It is important to appreciate that the K_{Ia} approach is, in general, not strictly accurate. Only in the highly idealised case where a semi-infinite crack propagates in an unbounded solid stressed by time-independent loads is the K_{Ia} approach exact[2]; the approach is not even exact with this geometrical configuration when the solid is subject to time-independent applied displacements[3]. Consequently, in general, the usefulness of the K_{Ia} procedure depends on its ability to make arrest predictions that are sufficiently accurate for practical purposes, although in many cases it is probably sufficient to demonstrate that the approach is conservative.

There are no wave reflection effects when a semi-infinite crack propagates in an unbounded solid and this problem can be investigated with the aid of the reflectionless stress intensity factor K_I^*, (2, 4, 5) which multiplied by an appropriate velocity term that equals unity when the crack tip velocity $v \to 0$, gives the dynamic stress intensity factor $K_I^{DYN}(v)$. Propagation is governed by the relation $K_I^{DYN}(v) = K_{ID}(v)$ where $K_{ID}(v)$ is the dynamic fracture toughness, which is presumed to be a monotonically increasing function of v. Arrest occurs when $K_I^{DYN}(v \to 0) = K_{ID}(v \to 0) = K_{ID}$ or when $K_I^* = K_{ID}$, and it has been demonstrated[1] that when a semi-infinite crack propagates in an unbounded solid stressed by time-independent loads, then the value of K_I^* at arrest (i.e. K_{ID}) is equal to the magnitude of the stress intensity factor as determined by a static LEFM analysis; in this case $K_{Ia} = K_{ID}$.

If wave reflections are neglected, the same approach can be used to study the propagation and arrest of an edge crack in a semi-infinite solid subject to a variety of loadings. This general model is appropriate for discussing the propagation and arrest of a crack in the pressure vessel of a water-cooled reactor when this is subjected to a hypothetical loss of coolant accident (LOCA). In such an accident, the emergency core cooling system (ECCS) injects water into the vessel, whereupon thermal stresses might enable a pre-existing defect at the vessel inner surface to propagate into the vessel wall. In this case, primarily because there are no free surfaces parallel to the crack propagation direction, one can develop arguments[6, 7] to show that it is reasonable to neglect wave reflection effects; the problem is therefore appropriate for investigation via the reflectionless stress intensity factor K_I^* approach. Thus if the simple K_{Ia} approach is to be practically useful in this type of situation, it is essential to demonstrate that the arrest predictions using the two approaches are in reasonable accord, or better still, that the K_{Ia} approach predictions are conservative.

It is against this general background that the present paper analyzes the propagation and arrest of an edge crack in a semi-infinite solid; the study extends and generalizes Melville's earlier investigation[5].

2. GENERAL THEORETICAL ANALYSIS

This section considers the Mode I propagation of an edge crack in a semi-infinite solid; the initial crack length is a_0 and the objective is to examine the situation when the crack propagates to a distance $(a_0+\epsilon)$ into the solid, i.e. the crack propagation distances is ϵ (Fig. 1a). Assuming that the applied loadings generate a tensile stress $p(a_0, x)$ ahead of the initial crack tip, the reflectionless stress intensity factor K_I^*, i.e. the product of K_I^{ST} and a "correction" factor g, is given[2, 4, 5] by the expression

$$K_I^* = gK_I^{ST} = \sqrt{\frac{2}{\pi}}\left[\int_{x=a0}^{x=a_0+\epsilon} \frac{\rho o(a_o, x)\mathrm{d}x}{\sqrt{(a_0+\epsilon)-x}}\right] \tag{1}$$

with x being measured from the crack mouth. It is important to appreciate that the derivation of relation (1) is based on an exact dynamic analysis, even though K_I^* is obtained by a purely static analysis.

The crack tip stress intensification factor K_I^{ST} for the extension ϵ is the same as that for an edge crack of length $(a_0+\epsilon)$ when pressures $p(a_0, x)$ are applied to its faces over a distance ϵ from its tip. Since this intensification factor exceeds that for a central crack of length $2(a_0+\epsilon)$ in an infinite solid, when pressures $p(a_0, x)$ are applied to its faces over a distance ϵ from both tips (Fig. 1b), it immediately follows that

$$K_I^{ST} > \frac{2}{\pi}\sqrt{(\pi(a_0+\epsilon))}\int_{x=a0}^{x=a_0+\epsilon} \frac{p(a_0, x)\,\mathrm{d}x}{\sqrt{((a_0+\epsilon)^2-x^2)}}. \tag{2}$$

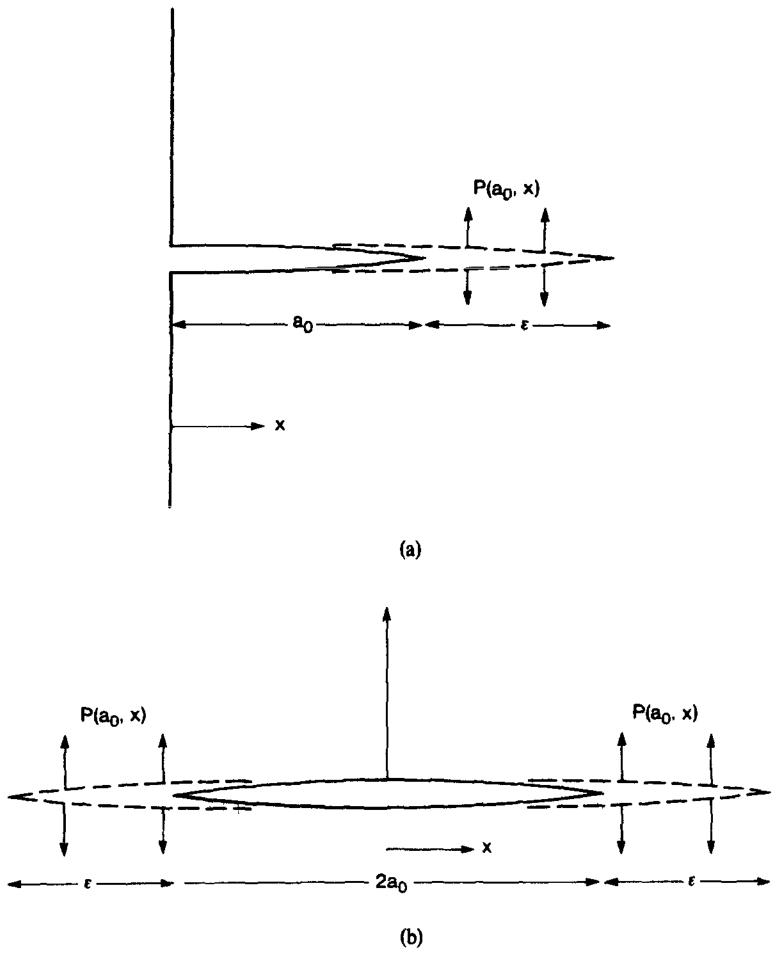

Fig. 1(a). The Mode 1 propagation of an edge crack of length a_0 in a semi-infinite solid. (b) The Mode 1 propagation of an internal crack of length $2a_0$ in an infinite solid.

Relations (1) and (2) give

$$K_I^{ST}-K_I^* > \sqrt{\frac{2}{\pi}} \int_{x=a0}^{x=(a_0+\epsilon)} \frac{[\sqrt{(2(a_0+\epsilon))}-\sqrt{((a_0+\epsilon)+x)}}{\sqrt{(a_0(a_0+\epsilon)^{2-}-x^2)}} p(a_0, x)\mathrm{d}x \tag{3}$$

and consequently K_I^{ST} is greater than K_I^* provided $p(a_0, x)$ is positive.

The crack arrests when the reflectionless stress intensity factor $K_I^*(\equiv gK_I^{ST})$ is equal to K_{ID}, whereupon it immediately follows that if K_{Ia} is the arrest value of K_I^{ST}, then $K_{Ia} > K_{ID}$ and the correlation factor $g < 1$; consequently the K_{Ia} approach is conservative in that it overpredicts the crack length at arrest. This conclusion generalizes that due to Melville[5] who considered the special case where the stress, in the crack's absence, is a linearly decreasing function of distance from the surface. It is valid whenever the applied loadings generate a tensile stress ahead of the original crack and is also applicable when a pressure is applied to the faces of the initial crack provided, of course, that they produce a tensile stress ahead of the crack.

As already indicated, Meville[5] has provided an example which illustrates this conclusion when external loads are applied to the solid; the next section provides simple examples where a pressure is applied to the faces of the initial crack.

3. EXAMPLES ILLUSTRATING THAT $K_{Ia} > K_{ID}$ WHEN A PRESSURE IS APPLIED TO THE CRACK FACES

As indicated in the previous section, in demonstrating that $K_{Ia} > K_{ID}$ when a pressure is applied to the faces of an edge crack in a semi-infinite solid, it is sufficient to demonstrate the condition for an internal crack in an infinite solid, with symmetric propagation occurring at each tip. This section therefore considers semi-infinite solid models but uses infinite solid analyses; the examples are chosen so that analytical solutions are readily attained, and, furthermore, they are such that arrest is possible, i.e. K_I^{ST} decreases as the crack extends.

As the first example, suppose that a semi-infinite solid contains an edge crack of initial length a_0 and concentrated forces P are applied at the crack mouth (Fig. 2). For this particular loading system, the tensile stress $p(a_0, x)$ ahead of the initial crack tip is

$$p(a_0, x) = \frac{2Pa_0}{\pi x\sqrt{(x^2-a_0^2)}} \tag{4}$$

where x is measured from the crack mouth, whereupon eqns (1) and (4) give the reflectionless

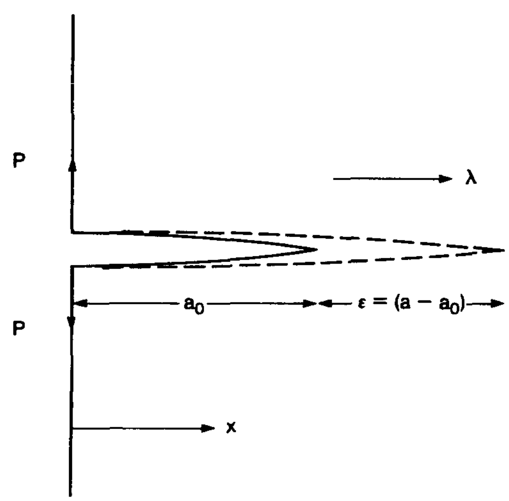

Fig. 2. The Mode 1 propagation of an edge crack in a semi-infinite solid due to the effect of applied forces P at the crack mouth.

stress intensity factor K_I^* for a crack extension ϵ as

$$K_I^* = \frac{4P}{\pi^{3/2}\sqrt{a_0}} \int_0^{\pi/2} \frac{d\theta}{\left[1+\frac{\epsilon}{a_0}\sin^2\theta\right]\sqrt{\left(1+\frac{\epsilon}{2a_0}\sin^2\theta\right)}} \tag{5}$$

i.e. K_I^* is given in terms of an elliptic integral of the third kind. Furthermore, relation (4) gives the crack tip stress intensification factor K_I^{ST} for an extension ϵ as

$$K_I^{ST} = \frac{2P}{\sqrt{(\pi(a_0+\epsilon))}}. \tag{6}$$

Assuming that the crack arrests when the reflectionless stress intensity factor $K_I^*(\equiv gK_I^{ST})$ is equal to K_{ID}, relations (5) and (6) immediately show that the arrest value of K_I^{ST}, i.e. K_{Ia} and the correction factor $g \equiv K_I^{ST}$ are given by the expression

$$\frac{K_{ID}}{K_{Ia}} = g = \frac{2}{\pi}\sqrt{\frac{a_0+\epsilon}{a_0}} \int_0^{\pi/2} \frac{d\theta}{\left[1+\frac{\epsilon}{a_0}\sin^2\theta\right]\sqrt{\left(1+\frac{\epsilon}{2a_0}\sin^2\theta\right)}}. \tag{7}$$

Figures 3 and 4 show respectively the variations of (K_{Ia}/K_{ID}) and g with crack jump length. For small crack jumps

$$\frac{K_{Ia}}{K_{ID}} = 1 + \frac{\epsilon}{8a_0} \qquad g = 1 - \frac{\epsilon}{8a_0} \tag{8}$$

while for large crack jumps

$$\frac{K_{Ia}}{K_{ID}} \sqrt{2} \qquad g \to \frac{1}{\sqrt{2}}. \tag{9}$$

The important conclusion for this particular loading pattern is that K_{Ia} increases and the correction factor g decreases as the crack jump length increases, i.e. K_{Ia} always exceeds K_{ID}.

Now consider the case where the semi-infinite solid contains an edge crack of initial length a_0 and concentrated forces P are applied to the crack faces at points that are at a distance $b(< a_0)$ from the crack mouth (Fig. 5). For this loading system

$$p(a_0, x) = \frac{2P\sqrt{(a_0^2 - b^2)}x}{\pi(x^2-b^2)\sqrt{(x^2-a_0^2)}} \tag{10}$$

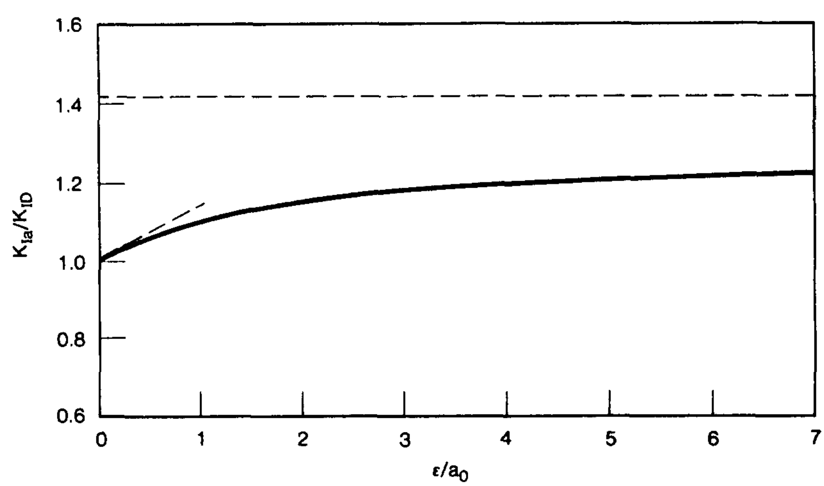

Fig. 3. (K_{Ia}/K_{ID}) as a function of crack-jump length (ϵ/a_0) for the model in Fig. 2.

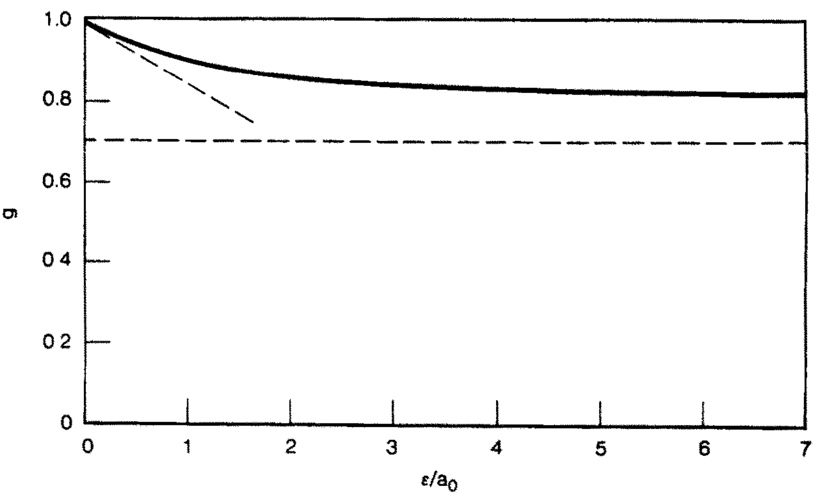

Fig. 4. The correction factor g as a function of crack-jump length (ϵ/a_0) for the model in Fig. 2.

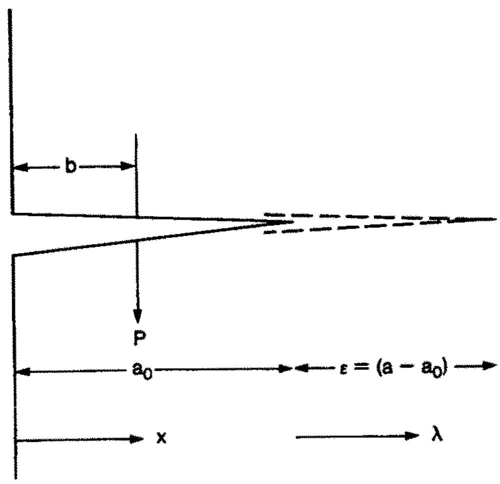

Fig. 5. The Mode 1 propagation of an edge crack in a semi-infinite solid due to the effect of applied forces P at the points shown.

where x is measured from the crack mouth. Relation (1) shows that the reflectionless stress intensity factor K_I^* for a crack extension ϵ is

$$K_I^* = \frac{4P}{\pi^{3/2}} \sqrt{\frac{a_0}{a_0^2 - b^2}} \int_0^{\pi/2} \frac{\left[1 + \frac{\epsilon}{a_0}\sin^2\theta\right] d\theta}{\left[1 + \frac{2a_0\epsilon \sin^2\theta}{(a_0^2 - b^2)} + \frac{\epsilon^2 \sin^4\theta}{(a_0^2 - b^2)}\right] \sqrt{\left(1 + \frac{\epsilon}{2a_0}\sin^2\theta\right)}}. \tag{11}$$

Furthermore, relation (10) gives the crack tip stress intensification K_I^{ST} for an extension ϵ as

$$K_I^{ST} = \frac{2P}{\sqrt{\pi}} \sqrt{\frac{a_0}{a_0^2 - b^2}} \frac{\sqrt{\left(1 + \frac{\epsilon}{a_0}\right)}}{\sqrt{\left(1 + \frac{2a_0\epsilon}{(a_0^2 - b^2)} + \frac{\epsilon^2}{(a_0^2 - b^2)}\right)}} \tag{12}$$

whereupon relations (11) and (12) show that the arrest value of K_I^{ST}, i.e. K_{Ia}, and the correction factor $g \equiv K_I^*/K_I^{ST}$ are given by the expression

$$\left[\frac{K_{ID}}{K_{Ia}} = g = \frac{2}{\pi}\right] \frac{\sqrt{\left(1+\frac{2a_0\epsilon}{(a_0^2-b^2)}+\frac{\epsilon^2}{(a_0^2-b^2)}\right)}}{\sqrt{\left(1+\frac{\epsilon}{a_0}\right)}} \int_0^{\pi/2} \frac{\left[1+\frac{\epsilon}{a_0}\sin^2\theta\right]\mathrm{d}\theta}{\left[1+\frac{2a_0\epsilon\sin^2\theta}{(a_0^2-b^2)}+\frac{\epsilon^2\sin^4\theta}{(a_0^2-b^2)}\right]\sqrt{\left(1+\frac{\epsilon}{2a_0}\sin^2\theta\right)}}. \tag{13}$$

For small crack jumps, relation (13) gives the same results as are expressed in relation (8).

Finally consider the case where the faces of the edge crack, again of initial length a_0, are subject to the uniform pressure P_N (Fig. 6). For this loading system

$$[p(a_0, x)] = \left[\frac{P_N x}{\sqrt{(x^2-a_0^2)}} - P_N\right] \tag{14}$$

where x is measured from the crack mouth. Relation (1) shows that the reflectionless stress intensity factor K_I^* for a crack extension ϵ is

$$K_I^* = \frac{2P_N\sqrt{a_0}}{\sqrt{\pi}}\left[\int_0^{\pi/2} \frac{\left(1+\frac{\epsilon}{a_0}\sin^2\theta\right)\mathrm{d}\theta}{\sqrt{\left(1+\frac{\epsilon}{2a_0}\sin^2\theta\right)}} - \sqrt{\frac{2\epsilon}{a_0}}\right] \tag{15}$$

and relation (14) gives the crack tip stress intensification factor K_I^{ST}, again for an extension ϵ, as

$$K_I^{ST} = \frac{2P_N}{\sqrt{\pi}}\sqrt{(a_0+\epsilon)}\sin^{-1}\left[\frac{a_0}{a_0+\epsilon}\right] \tag{16}$$

whereupon relations (15) and (16) show that the arrest value of K_I^{ST}, i.e. K_{Ia}, and the correction factor $g \equiv K_I^*/K_I^{ST}$ are given by the expression

$$\frac{K_{ID}}{K_{Ia}} = g = \frac{\sqrt{\frac{a_0}{a_0+\epsilon}}\left[\int_0^{\pi/2} \frac{\left[1+\frac{\epsilon}{a_0}\sin^2\theta\right]\mathrm{d}\theta}{\sqrt{1+\frac{\epsilon}{2a_0}\sin^2\theta}} - \sqrt{\frac{2\epsilon}{a_0}}\right]}{\sin^{-1}\left[\frac{a_0}{a_0+\epsilon}\right]}. \tag{17}$$

For small crack jumps, relation (17) again gives the same results as are expressed in relation (8).

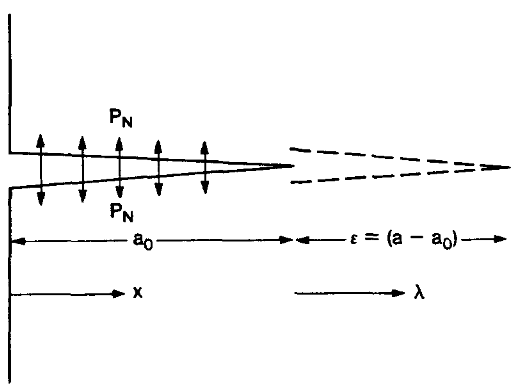

Fig. 6. The Mode 1 propagation of an edge crack due to the effect of a uniform pressure P_N applied to the faces of the crack when it is in its initial position.

4. DISCUSSION

The general theory in Section 2, illustrated by the specific examples in Section 3, shows both for the case where a pressure is applied to the crack faces and also for externally applied loads, that K_{Ia}/K_{ID} exceeds unity at arrest providing the loadings generate tensile stresses ahead of an edge crack in a semi-infinite solid. This implies that the simple K_{Ia} approach overestimates the crack length at arrest, i.e. the K_{Ia} approach is conservative from a safety viewpoint. This conclusion is applicable for all pressure and tensile stress distributions and generalizes the conclusion of Melville[5], who considered the specific case where the tensile stress, in the crack's absence, decreased linearly with distance from the solid's surface. In considering this conclusion in the context of the LOCA problem, it should be noted that the tensile stress in this case is generated thermally, while K_{ID} increases with depth into the pressure vessel wall. If, as seems appropriate, this situation is simulated by a model in which a tensile stress is generated by time-independent loads, the K_{Ia} approach is clearly conservative with regard to the prediction of the crack length at arrest.

It must be emphasized that the conclusion is limited in its applicability, since experimental test results, for example those obtained by Kalthoff[8], generally show a decrease of K_{Ia}/K_{ID} with crack-jump length. There are three possible explanations for this apparent difference: (a) the presence of free surfaces parallel to the crack may be having a significant effect on the interplay between K_I^{ST} and $K_I^\dagger$; (b) the loading in laboratory specimens is closer to being displacement rather than load controlled; (c) reflected waves may be reaching the crack tip and be affecting the arrest process. Kalthoff's experimental results[8] clearly show that wave reflections play a greater role in specimens (e.g. rectangular double cantilever beam) when free surfaces are close to the propagating crack, than in specimens (e.g. single edged-notched) where this is not the case. However, the effects of such surfaces in producing wave reflections is beyond the scope of this paper, whose main objective has been to focus on the conservatism of the simple K_{Ia} approach in situations where wave reflections are unlikely to have a major effect on the processes of dynamic crack propagation and arrest.

5. CONCLUSIONS

When an edge crack propagates in a semi-infinite solid due to applied loads or to pressures applied to the crack faces, the simple K_{Ia} approach is conservative in that it over-predicts the crack length at arrest, provided the loads produce a tensile stress ahead of the original crack.

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